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CONCEPT OF MEASURING AND CALCULATION SYSTEM, BASED ON ELECTRONIC CLINOMETERS TO DETERMINE THE GEOMETRY OF RAILWAY TRACK IN THE VERTICAL PLANE

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ABSTRACT: Geodetic investments support in the scope of modernization the existing sections of railways and building new ones is associated with the work of high accuracy. For this reason, surveying teams use the latest technological solutions, both in terms of hardware and software. Among the rail surveying systems dedicated for railway areas, there are trolleys equipped with GNSS satellite receivers. Unfortunately, the GNSS system is not sufficiently accurate in determining the height of the railway track. Therefore, the surveying work is often broken down into two stages. In the situational surveys total stations or GNSS receivers are used, and in the surveys of height, geometric levelling is applied. The article describes the possibility of using an electronic level in order to eliminate cumbersome levelling surveys. A clinometer used in the surveys firstly had been subjected to author's laboratory tests verifying its accuracy and allowing its calibration. During the field tests, the selected section of the track was measured using a level, and additional standard surveys were carried out using geometric precision levelling. A computational method was also suggested, allowing for the determination of the heights of points, based on the measured slopes. The method is based on integrating a continuous model of slopes, interpolated by a set of polygonal chains. The study conducted accuracy tests both of the computational method itself as well as combined, survey and computational method for determining the height.

1. INTRODUCTION

Management of the state railway network is the responsibility of PKP Polish Railway Lines S.A. Within the scope of rail infrastructure maintenance works, the company is required to conduct an ongoing review of its condition. The company carries out inspections and surveys, and records any faults or defects, as well as it conducts maintenance works, ongoing and main repairs. The purpose of this activity is to ensure safety on the railway lines and to maintain technical and operating parameters of a line at an adequate level. Among numerous works related to the maintenance of the lines, there is an inventory survey of the track geometry and location. Qualified diagnostic and surveying professionals perform works within the scope specified in the Instruction Id-14 (Instruction on carrying out surveys, testing and evaluation of tracks condition, 2005). The Instruction points to the necessity of carrying out surveys of the parameters related to the geometry of the track (i.e.: gauge, cant, unevenness of rails in horizontal and vertical planes). It also imposes an obligation to identify the location of the track in the horizontal and vertical planes in relation to the track axis reference points. These variables can be measured directly using a portable surveying equipment, as well as with indirect methods with the use of heavy surveying vehicles.

The process of upgrading of the existing sections of railway tracks and building new ones is supported by geodetic surveying teams at many stages. They are supposed to conduct both the inventory surveys, as well as geodetic support for the realization works. The most common surveying works performed in railway areas include: drawing maps for design purposes, inventory surveys to identify track geometry as well as realization surveys, including tamping machines surveying services. An essential element associated with the works which are aimed at determining the geometry of the track is executing a longitudinal profile or its updating. The data from these surveys is often used to make adjustments of track axes (Instruction D-19, 2000).

The surveying works can use a variety of technological solutions. The most precise and efficient ones, allowing simultaneous surveys of multiple geometrical parameters of the track, may include track measurement *trolleys*. These are multi-sensor devices providing information on ground clearance, cant and track stationing. Additionally equipped with geodetic equipment they guarantee the possibility of determining the location of the track in the adopted spatial coordinate system. Recently, designers of the measuring trolleys equip them with GNSS precision satellite receivers. Also at AGH University of Science and Technology in Kraków such a trolley was built (Strach, 2003). The conducted measurement tests indicate that the technique RTK GNSS allows to specify a railroad track geometry with sufficient accuracy only in the horizontal plane. GNSS measurements with the antenna placed on the trolley are not able to provide sufficiently precise heights of the observed points. Ordinates of the tracks under inventory should be determined by means of geometric levelling. Geodetic network points of height along the track are used to engage in surveys. Great help in carrying out the surveys are points stabilized on traction current pylons (Fig. 1), acting as both the reference points and the railway geodetic control network. Breakdown of the works into two independent technological stages (situational and height) prolongs the time of carrying out surveys and increases the costs of their implementation. The device, which can replace the tedious levelling method is a precise electronic level (clinometer) mounted on the measurement trolley. The clinometer should be placed on the construction beam of the trolley parallel to the axis of the track. This positioning will allow the measurement of longitudinal rail slopes. Before deciding to expand the track measurement trolley with the aforementioned device, appropriate tests and analysis of the results were carried out. The tests of the constructed electronic clinometer (Fig. 2) were conducted in the laboratory and in the field. Precise model surveys were conducted allowing the analysis of accuracy of the obtained results from the surveys with the clinometer. Also a mathematical algorithm was proposed allowing to determine the height of the points of track grade line, based on the observed slopes.

Fig. 1. Track axis reference point and at the same time railway geodetic network point fixed to a traction current pylon

Fig. 2. Electronic level during the field tests

2. MATHEMATICAL PRINCIPLES OF DETERMINING THE HEIGHT OF GRADE LINE POINTS BASED ON THE OBSERVED SLOPES

The result of the carried out surveys are sets of two values: the slopes of the track and the mutual distances of points at which the records of the slopes were made. Additionally, opposite the selected traction pylons, by the use of track reference points stabilized on them, the heights of the tracks shall be specified. Based on these values, the heights of the track should be determined at the slope observation points. Track reference points on the pylons are arranged along the track every 60 – 70 meters on straight sections, while on curved sections the density of the pylons is higher. Observations carried out with the level are performed while the trolley is not moving, which takes place every few meters. As a result, based on the height of the track transferred from the pylon, the heights of over a dozen points on average should be determined, where the slope was observed. The process is repeated periodically, for sections between successive traction current pylons, which at all times ensure the initialization of the determination of the track height.

To solve the task, the method of integration of functions describing the slopes may be applied, which was previously used in (Gocał, 1981; Gocał, Lenda, 2003). The idea of the method goes down to the following observations. The observed slope at a given point Pi is a boundary position of the secants of the track grade line, connecting successive pairs of points, i.e. Pi-1 - Pi and Pi - Pi +1, thereby constituting the slope of the tangent to the grade line at this point. The gradient of the tangent to the curve k at a given point is determined by setting the value of its derivative at this point. Determination of the value of the function k itself, that is the height of the grade line points, will be subject to a reverse process, i.e. integrating of the functions representing a set of slopes of the tangents. For this purpose, an approximating function should be fitted into a set of slopes observed with the level, whose primary function, assuming the appropriate initial conditions, shall appoint the searched heights. From the standpoint of the accuracy of the results obtained in this manner, the type of function describing the slope is essential. In the above-mentioned studies approximating polynomials were used for this purpose. In general, the polynomials can be represented in the interpolating or approximating form. In the case of a greater number of points, interpolation requires the use of polynomials of higher degrees, whose determining process, with an increasing degree, is fast becoming an ill-conditioned numerical task (Fortuna, Macukow, Wąsowski, 2001; Cheney, Kincaid, 2006), generating significant distortions of the resulting function . Approximation in turn, made with low-degree polynomials, does not allow a good fit of a function to the set of slopes, which also has an adverse effect on the accuracy of the designated heights of the grade line. However, due to the fact that the track grade line does not demonstrate large curvatures, good accuracy can be achieved even with the replacement of point model of the slope with the simplest continuous model in the form of a polygonal chain. This paper presents experimental results achieved using an algorithm based on such a method.

Each section connecting consecutive points of the observed slopes is described with the equation:

$$F_i^{'}(x) = a_{1i} + a_{2i}x \tag{1}$$

Determination of the function describing the track grade line goes down to integrating all of such equations:

$$\int F_i^{'}(x) = F_i(x) = a_{1i}x + \frac{1}{2}a_{2i}x^2 + c_i \tag{2}$$

and to defining the appropriate constants of integration c_i. The value of the first of these can be determined using the known height of the track H_0, referred to the first point located opposite the reference point on the traction current pylon.

$$c_0 = H_0 - \left(a_{10} x_0 + \frac{1}{2} a_{20} x_0^2 \right) \tag{3}$$

The first section of the function $F_0(x)$ connects the two initial grade line points, and therefore it enables to calculate the height of the next point at which the slope was observed. This height will be the basis to designate the constant of integration of the next section of the function $F_i(x)$ in the same way as for the formula (3). The whole algorithm is repeated until reaching the ordinate of the next traction current pylon equipped with track reference point, from which the exact height will be transferred onto the track. From this point the algorithm is invoked again, setting the grade line height for the next section located between the track reference points.

3. METHODOLOGY OF THE RESEARCH AND RESULTS OF THE PRECISION ACCURACY TESTS

In order to verify the accuracy achieved by the proposed method, tests were performed in three categories: the accuracy of the surveying method (electronic level), the accuracy of the computational method and the total accuracy of the system.

3.1 Accuracy of the electronic level

The basis for the realization of experimental studies on the use of clinometers in determining the height of the track was to find a sufficiently precise electronic level. The autors used an electronic clinometer (Fig. 1), built for the purpose of levelling of machinery and technological equipment. It is a single-axis analogue inclinometer made by the company VTI Technologies Oy, mounted on the system SCA103T. In accordance with the technical specification, the device operates with a resolution of 0.001 °, and the sensor measuring range falls within ± 15 ° (www.vti.fi, 2011).

Each instrument is characterized by its own distribution of errors, knowledge of which becomes necessary in the case of high-precision measurements. The verification of accuracy and calibration of the electronic level were carried out in geodetic metrology laboratory of AGH in Kraków.

In laboratory tests, a precise electronic total station Leica TC2002 was used, with the accuracy of measuring vertical angles of 0.5". The level was attached to the total station telescope set horizontally (Fig. 3). The idea of the test consisted of recording the consecutive pairs of readings from the sloping of the level and the total station telescope. After each survey, the telescope was rotated with an interval equal to approximately 0.5 mm / m, wchich was read from the level.

The values recorded from both of the devices were normalized to be able to compare and evaluate the accuracy of the clinometer. The graph illustrated in Figure 4 presents the characteristics of the clinometer operation compared with the aforementioned total station.

With the full range of work of the level (-10 ÷ +10 mm/m) the differences between the indications of these two devices reach 0.31 mm/m. The values of the differences in readings from both devices enabled the calculation of the calibration corrections for the controlled level. Once they are included in the results, we can conclude that the accuracy of the clinometer is no worse than 0.1 mm/m.

Fig. 3. Electronic level with electronic total station Leica TC2002 during testing in geodetic metrology laboratory of AGH University of Science and Technology

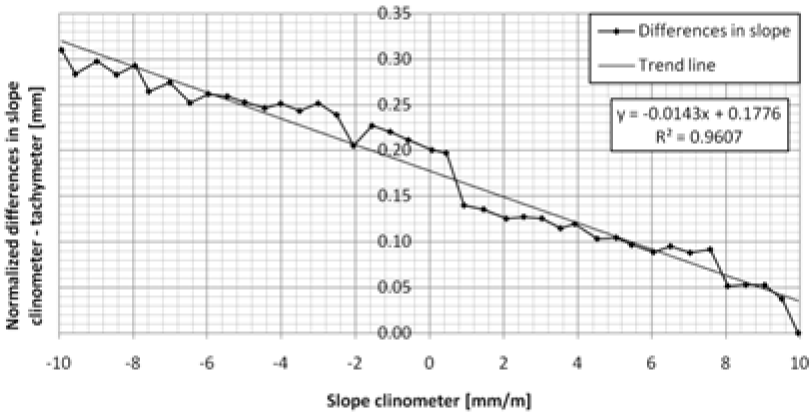

Fig. 4. Clinometer work characteristics compared with total station Leica TC2002

3.2 Accuracy of the computational method

The accuracy of the computational method was tested based on the model data generated on the base of levelling surveys of the track grade line. Precise levelling provided a set of grade line heights of the points with known mutual distances along the track. Spline function was fitted into this set, allowing the replacement of a point model with a smooth model of a continuous, and also a minimum total curvature. For the analyzed case, the so called *natural spline function* will have the desirable properties, which according to the Holladay's theorem (Ahlberg, Nilson, Walsh; 1967) offers the best smoothness. To create a continuous model, a function in the form of B-spline (Kiciak, 2000) was used:

$$S_i(x) = \sum_{i=0}^{n-4} d_i N_i^3(x) \qquad x_i = \{x_0, .., x_n\} \tag{4}$$

where: d_i = coefficients of linear combination of basis functions N_i, defined by the formula Mansfield-De Boor-Cox (Kiciak, 2000)

Then, at the height measurement points (interpolation), tangents to this function were determined, specifying their slope, which are a model value of the measurements which should be made in practice using the level. The slopes were read from the model with an accuracy appropriate for the described level, i.e. at the level of 0.1 mm/m.

The slopes obtained in this way were then used to determine the height of the track grade line points according to the proposed method. The results were compared with the heights of the grade line selected to create a model (from levelling). In this way, the quality of the mathematical method operating on the measurement data with a given accuracy was defined.

The tests were conducted for three selected sections measured by precise levelling of the track, each consisting of twenty points. The sections (of ten meter length), which were selected for the test, with the vertical scale increased respectively (100-fold), have been shown in Figure 5. The first of them (m1) has a relatively constant slope, the second one (m2) – has a variable slope and small changes in curvature, and the third one (m3) - has a variable slope and significant changes in curvature.

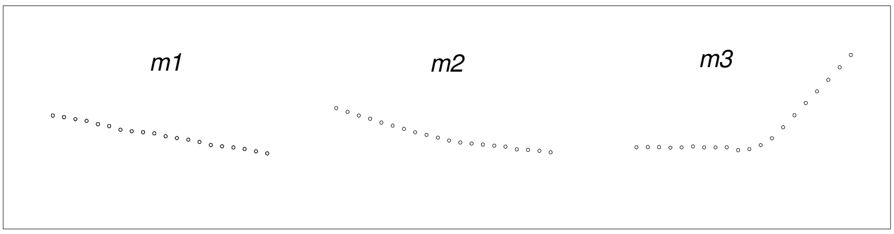

Fig. 5. Sections selected for testing for the study of the accuracy of the computational method. The vertical scale in relation to the horizontal scale: 100: 1. Horizontal distance between points equals 10 [m]

The results of the carried out tests have been compiled in Table 1.

Tab. 1. Determination of the accuracy of the computational method for the model data. The differences in heights dH from levelling, and determined by integrating the function describing the slope

	Accuracy of determining height *dH [mm]*		
	m1	*m2*	*m3*
1	0.0	0.0	0.0
2	0.6	0.3	-0.3
3	-1.1	-0.7	0.6
4	2.7	1.5	-1.0
5	-3.2	-1.1	-1.5
6	3.5	0.8	2.4
7	-3.3	0.3	-1.7
8	0.3	-0.7	-1.2
9	-0.5	0.4	4.3
10	2.3	0.8	-5.1
11	-1.7	-1.0	-2.0
12	0.1	-0.4	-1.8
13	0.2	0.3	-4.4
14	1.4	0.3	-0.5
15	-2.4	-0.5	-0.4
16	0.3	2.9	0.2
17	0.3	-3.1	0.0
18	0.4	1.5	-1.5
19	-0.2	0.4	0.2
20	0.0	0.8	-0.4
***dH* max.**	3.5	3.1	5.1
***dH* mean**	1.2	0.9	1.5

3.3 The total accuracy of the method

In order to obtain a total measurement and computational estimate of the accuracy of the proposed system, surveys were conducted on a section of the track line Warsaw – Kraków, of the route Słomniki - Niedźwiedź at the stationing of 291.100 ÷ 291.381. The observations were carried out by two independent methods: using a precise geometric levelling, which is the reference method, and using the electronic level. The level was applied twice, independently, on the surface of the rail head, parallel to its axis. The places of the level application were the previously determined and surveyed (with the use of geometric levelling) cross-sections on the track. Geometric levelling was carried out with the use of precise leveller DNA03 with a precise digital levelling rod. This set has an accuracy of 0.3 mm/km. In the reported surveys closed levelling traverse was applied. One of the reference points was used to determine the height. The surveyed levelling traverse is characterized by an error of 0.8 mm for the length of 800 m. As a result of the surveys, the heights of all of the track reference points were determined. Also the heights of the track opposite the reference points and in cross-sections every 2 meters were determined.
The tests were conducted in an analogous manner as described above, on the same track sections (Fig. 5). For each of them the heights of the grade line points were calculated based on the integration of the functions fitted into the surveyed slopes, and afterwards they

were compared with the heights obtained by precise levelling. The differences between the two methods have been presented in Table 2.

Tab. 2. Determination of total, measurement and computational accuracy of the method. The differences in heights dH from levelling, and determined by integrating the function describing the slope

	Accuracy of determining height *dH [mm]*		
	m1	***m2***	***m3***
1	1.1	17.0	-14.7
2	2.9	8.9	-14.5
3	-9.3	3.6	-3.0
4	-6.0	16.4	-10.6
5	-3.4	10.2	2.4
6	-15.3	3.2	14.9
7	-13.6	7.8	14.5
8	-7.8	-9.5	-7.8
9	8.0	-11.1	1.8
10	-5.7	-16.7	2.8
11	7.4	-9.0	6.4
12	-4.0	-15.1	15.7
13	12.5	1.6	5.3
14	9.0	9.9	-7.7
15	7.1	12.7	-11.2
16	-8.7	5.4	-15.1
17	13.9	-3.9	-3.6
18	4.2	-9.8	6.5
19	-3.3	-17.5	-4.5
20	9.5	-13.2	-7.3
***dH* max.**	15.3	17.5	15.7
***dH* mean**	7.6	10.1	8.5

3.3.1. Accuracy of the level

The obtained test results allow the conclusion that the analyzed level enables surveying with an accuracy of 0.1mm/m. In the geodetic metrology laboratory of AGH in Kraków, a laser interferometer HP 5529 is also available. The accuracy of the surveys performed with this device reaches the µm/m range. In the future it is planned to calibrate the level using the aforementioned interferometer.

3.3.2. The accuracy of the computational method

The proposed computational method, assuming a slope measurement accuracy of 0.1 mm/m, achieved satisfactory results (Table 1). Maximum deviations for the two tested sections were at the level of 3 mm, for the third one - 5 mm. The obtained results are within the standards of measurement accuracy, which are specified in Instruction D-19, § 6 p.96). It includes a clause that the heights of the points of the track system and terrestrial fittings of underground utilities should be determined by total station measurements, and additionally with geometric levelling method. Deviation between the ordinates of the two methods of surveying cannot exceed ± 1 cm. Deviations greater than this value make these surveys to be repeated. For the analyzed set of observations the average deviations in no case exceed the value of 1.5 mm. For all the models, in the areas of the occurrence of maximum deviations, oscillations of their values were recorded, based on the frequent change of the reference point.

3.3.3.The overall measurement and calculation accuracy

The overall measurement and computational accuracy of the test results is often at an unsatisfactory level. Permissible deviation limits at some points are exceeded by 1.5 times. Deviations are distributed randomly, and their occurrence is associated with insufficiently accurate way of applying the level to the rail. The device has the accuracy of 0.1 mm / m. However, its unequal applying to the upper surface of the railhead resulted in inaccuracies, already apparent during the survey. Significant differences in the slopes from double readings of the level at each point were observed. Placing the level on a measuring trolley which ensures repeatable application of the level to the track, eliminates the problem.

4. CONCLUSIONS AND FUTURE RESEARCH PLANS

The used level meets accuracy requirements necessary for the implementation of the works on the railroad tracks. Unfortunately, the way it is applied to the rail did not assure obtaining the required accuracy. Also, for practical reasons and due to the nuisance of its application, modification should be introduced in its construction and in the manner of conducting surveys. Currently, intensive works on improving both the equipment and the measurement technology are being carried out.

In the scope of the proposed computational method, it should be noted that for the total tests, there were no places of signs oscillations of deviations with the largest values, occurring for the tests of the computational method. It can therefore be estimated that these oscillations are not necessarily related to the operation of the proposed computational method. They may be partially due to the inaccuracy of the model, created for the earlier tests using spline functions. The results from Table 1 evaluating the accuracy of the computational method itself, as disturbed by an additional modelling error, should therefore be evaluated as more optimistic. They can be treated as top-down estimate of the accuracy offered by the proposed computational method.

The use of a polygonal chain allows for obtaining good accuracy of the determined heights at track reference points of the slopes. In order to determine the height at random points of the track grade line, non-linear functions should be used. For this purpose, the authors provide for the use of spline functions for which they are currently carrying out accuracy tests. This method is also likely to allow an overall increase in computational accuracy.

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